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Remark on the string field for a general configuration of N D-instantons

J Klusoň

Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37, Brno, Czech Republic

E-mail: klu@physics.muni.cz

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Abstract
In this paper we would like to suggest a matrix form of the string field for any configuration of N D-instantons in bosonic string field theory.

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1. Introduction

Renewed attention has been paid to Witten's [1] cubic bosonic open string field theory, following Sen's [3] conjectures that this formalism can be used to give an analytic description of D25-brane decay in bosonic string theory (for a review and an extensive list of references, see [2]). It seems to be possible that string field theory could give very interesting information about the nonperturbative nature of string theory and consequently about M theory. For this reason it seems interesting to study the relation between string field theory and matrix theory [4], which is the most successful nonperturbative definition of M theory. For example, recent progress in the vacuum string field theory [5–9, 11–16, 18–21] suggests that the string field theory could be useful for better understanding of the basic fabric of the string theory.

In order to find a relation between matrix theory and string field theory, it would perhaps be useful to study the non-Abelian extension of string field theory as well. As was stressed in the original paper [1], the non-Abelian extension of string field theory can be very easily implemented into its formalism by introducing Chan–Paton factors for various fields in the string field theory action and including the trace over these indices. In modern language this configuration corresponds to N coincident D25-branes.

In our previous paper [22] we proposed a generalized form of the string field theory action that was suitable for description of the general configuration of D-instantons. We have formulated this theory in a pure abstract form following the seminal paper [1]. In order to support further our proposal, we think that it would be desirable to have an alternative formulation of the generalized matrix string field theory action which would allow us to perform more detailed calculation. In particular, it would be nice to have a matrix generalization of the action written in the conformal field theory (CFT) language [23,24]. In this paper we suggest a possible form of the matrix-valued string fields that will be building blocks for the matrix CFT formulation of the string field action. We present a compact form of this matrix-valued string field. We shall study its operator product expansion (OPE) with the open string stress–energy tensor and we shall show that in order for any general component of the string field to have a well defined conformal dimension the background configuration of N D-instantons (n this paper we shall discuss D-instantons only; the extension to Dp-branes of any dimension is trivial) must obey one particular condition that can be interpreted as a requirement that the background configuration of D-instantons is a solution of the equation of motion arising from the low-energy action for the D-instanton matrix model. In our opinion this situation is similar to the fact that consistent string field theory should be formulated using CFT, that forces the background field to obey the equation of motion.

Then we extend our analysis to the case of infinitely many D-instantons and we show that the well known non-Abelian configuration can be very easily included in our formalism. In particular, we find such a matrix form of the string field and hence vertex operators that precisely corresponds to the string field theory formulated around a noncommutative D-brane background [25,26].

In conclusion we outline our results and suggest extension of this work. In particular, it will be clear from this paper that the extension of our approach to the supersymmetric case can be very easily performed.

2. String field theory in the CFT formalism

In this section we review basic facts about bosonic string field theory, following mainly [2,3]. Gauge-invariant string field theory is described by the full Hilbert space of the first quantized open string including b, c ghost fields subject to the condition that the states must carry ghost number 1, where b has ghost number -1, c has ghost number 1 and the $SL(2,C)$-invariant vacuum $|0\rangle$ carries ghost number 0. We denote by $\mathcal{H}$ the subspace of the full Hilbert space carrying ghost number 1. Any state in $\mathcal{H}$ will be denoted as $|\Phi\rangle$ and the corresponding vertex operator $\Phi(x)$ is the vertex operator that creates state $|\Phi\rangle$ from the vacuum state $|0\rangle$

$$|\Phi\rangle = \Phi(x)|0\rangle. \tag{2.1}$$

Since we are dealing with open string theory, the vertex operators should be put on the boundary of the world-sheet.

The open string field theory action has a form

$$S = \frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\langle I\circ\Phi(0)Q_B\Phi(0)\rangle + \frac{1}{3}\langle f_1\circ\Phi(0)f_2\circ\Phi(0)f_3\circ\Phi(0)\rangle\right), \tag{2.2}$$

where g_0 is the open string coupling constant, Q_B is the BRST operator and $\langle\,\rangle$ denotes the correlation function in the combined matter ghost CFT. I, f_1, f_2, f_3 are conformal mappings, the exact form of which is reviewed in [2], and $f_i\circ\Phi(0)$ denotes the conformal transformation of $\Phi(0)$ by f_i. For example, for Φ a primary field of dimension h, then $f_i\circ\Phi(0) = (f_i'(0))^h\Phi(f_i(0))$.

We can expand any state $|\Phi\rangle\in\mathcal{H}$ as

$$|\Phi\rangle = \Phi(0)|0\rangle = (\phi(y) + A_\mu(y)\alpha_{-1}^\mu + B_{\mu\nu}(y)\alpha_{-1}^\mu\alpha_{-1}^\nu + \cdots)c_1|0\rangle = \sum_\alpha \phi^\alpha(y)\Phi_\alpha(0)|0\rangle, \tag{2.3}$$

where coefficients $\phi^\alpha(y)$ in front of basis states $|\Phi_\alpha\rangle$ of $\mathcal{H}$ depend on the centre-of-mass state coordinate y and where index α labels all possible vertex operators of ghost number 1.

As we think of the coefficient functions $\phi_\alpha(y)$ as spacetime particle fields, we call $|\Phi\rangle$ a string field [2]. The vertex operator $\Phi(z)$ defined above is also called a string field.

The previous action describes string field theory living on one single D25-brane. In order to describe a configuration of N coincident D25-branes we equip the open string with Chan–Paton degrees of freedom so that coefficient functions become matrix valued and so does $|\Phi\rangle$. In the following we restrict ourselves to the case of N D-instantons where strings obey Dirichlet boundary conditions in all dimensions and where coefficient functions are $N \times N$ matrices without any dependence on y. We shall write such a string field as $|\hat{\Phi}\rangle$ and call it mostly in the text a *matrix-valued string field*, keeping in mind that this is an $N \times N$ matrix where each particular component $|\hat{\Phi}\rangle_{ij}$ corresponds to the string field that describes the state of the string connecting the ith D-instanton with the jth D-instanton.

As we mentioned in the introduction, it would be interesting to have a formulation of the string field action for any configuration of D-instantons. While some progress in this direction was made in [22], we would like to find such a formulation of the action based on the CFT description. As the first step in searching for such a string field theory action we propose generalized matrix-valued vertex operators carrying CP factors that describe any configuration of D-instantons. We shall discuss this approach in the next section.

3. String fields for N D-instantons

We propose the form of the matrix-valued string field which in our opinion provides a description of the general configuration of N D-instantons in the bosonic string field theory. D-instantons are characterized by the strings having Dirichlet boundary conditions in all dimensions $y^I, I = 1, \ldots, 26$. Let us consider the situation with N D-instantons placed in general positions. This background configuration is described by matrices (see, for example [30, 31])

$$Y^I = \begin{pmatrix} y_1^I & 0 & \ldots & 0 \\ 0 & y_2^I & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & y_N^I \end{pmatrix}, \qquad I = 1, \ldots, 26, \tag{3.1}$$

where y_i^I labels the coordinate of the ith D-instanton. Moreover, the configuration (3.1) corresponds to the solution of the equation of motion of the low-energy matrix model effective action and, as we shall see, some consistency requirements that will be posed on the matrix-valued string fields also imply that the background configuration of N D-instantons (3.1) should have this form.

It is well known that the string stretching from the ith D-instanton to the jth D-instanton has an energy proportional to the distance between these two branes. More precisely, in the CFT language with the background corresponding to N D-instantons in the general position the ground state of the string going from the ith D-instanton to the jth D-instanton is described by the vertex operator[1]

$$|ij\rangle \equiv u_{ij}(z=0)c(0)|0\rangle \equiv c(0)\exp\left(\frac{\mathrm{i}(y_i^I - y_j^I)}{2\pi\alpha'} g_{IJ} X^J(0)\right)|0\rangle, \qquad i, j = 1, \ldots, N, \tag{3.2}$$

with $|0\rangle$ being the $SL(2, C)$-invariant vacuum state. In the previous expression g_{IJ} is a flat closed string metric $g_{IJ} = \delta_{IJ}$ with signature $(+\cdots+)$. Let us consider some state of ghost

[1] Because we implicitly presume that U is normal ordered we shall not write the symbol of normal ordering. For simplicity, we shall also consider the dependence of the world-sheet fields $X^I(z)$ on the holomorphic coordinate z only.

number 1 from the first quantized Hilbert space of the open string that does not depend on the zero mode of $X^I(z)$, which means that $\Phi_\alpha = \Phi_\alpha(\partial X, c, b)$ commutes with u_{ij} given above. The index α labels all possible vertex operators of ghost number 1. Then any string field corresponding to the string going from the ith D-instanton to the jth D-instanton can be written in CFT language in a similar form as in (2.3)

$$|\hat{\Phi}\rangle_{ij} = \sum_\alpha A_{ij}^\alpha \Phi(0)_\alpha u_{ij}(0)|0\rangle, \tag{3.3}$$

where $(A)_{ij}^\alpha$ is the analogue of $\phi^\alpha(y)$ in (2.3). Roughly speaking, matrix $A^\alpha \in U(N)$ contains information on which string from the collection of all possible N^2 strings of the system of N D-instantons (or more precisely, with which amplitude of probability) is excited in a given state characterized by the world-sheet operator $\Phi_\alpha(z)$. In the following we restrict ourselves to one particular CFT operator $\Phi_\alpha(z)$ and its corresponding A^α. For this reason we omit the index α from our formulae. In spite of this fact we shall still call $\hat{\Phi}$ a string field since it describes the whole system.

From the previous analysis it is clear that any string field is $N \times N$ matrix that in more detailed description has a form

$$\hat{\Phi}(0) = \begin{pmatrix} A_{11}u_{11}(0) & A_{12}u_{12}(0) & \dots & A_{1N}u_{1N}(0) \\ A_{21}u_{21}(0) & A_{22}u_{22}(0) & \dots & A_{2N}u_{2N}(0) \\ \dots & \dots & \dots & \dots \\ A_{N1}u_{N1}(0) & \dots & A_{N,N-1}u_{N,N-1}(0) & A_{NN}u_{NN}(0) \end{pmatrix} \Phi(0). \tag{3.4}$$

We would like to argue that this expression can be written in a more symmetric form. Our proposal is that the generalized matrix-valued string field can be written as

$$\hat{\Phi}(z) = U(z)(A)\Phi(z), \tag{3.5}$$

where we define the $N \times N$ matrix operator $U(z)(\cdot)$ that is a function of the matrices Y^I and the world-sheet fields $X^I(z)$. We shall show that for the background given (3.1) the operator (3.5) reduces to (3.4). Now we should clarify some points regarding our proposal. First of all, we consider a static configuration of N D0-instantons in a general position. As was said above and as is well known, such a background is described by matrices Y^I. CFT description of this system leads to the emergence of winding charge as we argued above and as is well known. This approach can be easily included in the CFT description of Witten's open string field theory as can be seen in the beautiful example of calculation of the mass of the D0-brane in [3]. In fact, our approach can be seen as rewriting this approach in a more symmetric and general form, that in our opinion could be more appropriate for string field theory formulation. As we shall see, we can also start with the general matrices Y^I and then using arguments of well defined conformal transformation of vertex operators obtain a condition that Y^I should obey. We shall also be able to generalize this approach to the case of infinite-dimensional matrices Y^I, that will lead to the natural emergence of the string field theory for the CFT background corresponding to a higher-dimensional D-brane as well known from matrix theory (for example [4]). For these reasons we believe that our generalization could be useful.

We could also proceed in a different way in order to describe a general configuration of D-instantons. We can start with the background of N D-instantons at the origin and try to find a solution of the string field theory equation of motion corresponding to the marginal deformation of this configuration to the general positions of D-instantons. However, as shown in [32], this is a very difficult problem when an infinite number of component fields in the string field should have nonzero expectation value. Then we see that a general configuration of D-instantons in string field theory should be described by a very complicated object that contains infinite components. On the other hand, when we start from the initial configuration of D-instantons that is described by all string matrix values, then using the standard construction of the new BRST operator (see, for example, [5, 10, 11]), we can conclude that the new BRST operator describing string fluctuations around the new background configuration of D-instantons should be matrix valued. Then, generalizing arguments of [32], we can argue that there should exist a complicated, probably singular field redefinition that maps the new BRST operator to the original one and where the new configuration of D-instantons is described by the CFT operators whose forms were revived above and that measure the corresponding winding charge of the strings going from the ith D-instanton to the jth D-instanton. In our proposal we presume that such a field redefinition has already been performed so that the BRST operator is diagonal and the background configuration of D-instantons is completely coded in the form of the new matrix-valued operator.

To see this, we must first explain the meaning of the expression $U(z)(A)$. This expression simply corresponds to the expansion of the exponential function and successive action of the commutators on matrix A. More precisely

$$U(A)_{ij} = \exp\left(\frac{\mathrm{i}}{2\pi\alpha'}[Y^I, A]g_{IJ}X^J(z)\right)_{ij} = A_{ij} + \frac{\mathrm{i}}{2\pi\alpha'}[Y^I, A]_{ij}g_{IJ}X^J(z) + \frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^I, [Y^K, A]]_{ij}g_{IJ}g_{KL}X^J(z)X^L(z) + \cdots. \tag{3.6}$$

The second term in (3.6) for Y^I given in (3.1) is equal to

$$\frac{\mathrm{i}}{2\pi\alpha'}[Y^I, A]_{ij}g_{IJ}X^J(z) = \frac{\mathrm{i}}{2\pi\alpha'}[y^I_i\delta_{ik}A_{kj} - A_{ik}y^I_k\delta_{kj}]g_{IJ}X^J(z) = \frac{\mathrm{i}}{2\pi\alpha'}[y^I_i - y^I_j]A_{ij}g_{IJ}X^J(z), \tag{3.7}$$

where there is no summation over i, j. In the same way the third term in (3.6) gives

$$\begin{aligned}
&\frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^I, [Y^K, A]]_{ij}g_{IJ}g_{KL}X^J(z)X^L(z) \\
&= \frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^I_{im}, [y^K_m - y^K_j]A_{mj}]g_{IJ}g_{KL}X^J(z)X^L(z) \\
&= \frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2(y^I_m\delta_{im}(y^K_m - y^K_j)A_{mj} - (y^K_i - y^K_m)A_{im}\delta_{mj}y^I_j) \\
&\quad\times g_{IJ}g_{KL}X^J(z)X^L(z) \\
&= \frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2(y^I_i(y^K_i - y^K_j)A_{ij} - (y^K_i - y^K_j)y^I_jA_{ij})g_{IJ}g_{KL}X^J(z)X^L(z) \\
&= \frac{1}{2}A_{ij}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2(y^I_i - y^I_j)g_{IJ}X^J(z)(y^K_i - y^K_j)g_{KL}X^L(z) \\
&= \frac{1}{2}A_{ij}\left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_i - y^I_j)g_{IJ}X^J(z)\right)^2,
\end{aligned} \tag{3.8}$$

where from the fourth row there is no summation over i, j. To show that (3.5) really corresponds to (3.4) for Y^I given in (3.1) we use the proof by mathematical induction. Let us presume that the following relation is valid for any P:

$$\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^P[Y^{I_1}, [Y^{I_2}, \ldots, [Y^{I_P}, A]]]_{ij}g_{I_1J_1}g_{I_2J_2}\ldots g_{I_PJ_P}X^{J_1}(z)X^{J_2}(z)\ldots X^{J_P}(z) = A_{ij}\left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_i - y^I_j)g_{IJ}X^J(z)\right)^P. \tag{3.9}$$

Then for $P + 1$ we have

$$\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P+1}[Y^{I_1},[Y^{I_2},\ldots,[Y^{I_{P+1}},A]]]_{ij}g_{I_1J_1}g_{I_2J_2}\ldots g_{I_{P+1}J_{P+1}}X^{J_1}(z)X^{J_2}(z)\ldots X^{J_{P+1}}(z)$$
$$= \frac{\mathrm{i}}{2\pi\alpha'}\left[Y^K_{ik}A_{kj}\left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_k - y^I_j)g_{IJ}X^J(z)\right)^P - \left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_i - y^I_k)g_{IJ}X^J(z)\right)^P A_{ik}Y^K_{kj}\right]g_{KL}X^L(z)$$
$$= \left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P+1}\big(y^K_i A_{ij}\big((y^I_i - y^I_j)g_{IJ}X^J(z)\big)^P - y^K_j A_{ij}\big((y^I_i - y^I_j)g_{IJ}X^J(z)\big)^P\big)g_{KL}X^L(z)$$
$$= A_{ij}\left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_i - y^I_j)g_{IJ}X^J(z)\right)^{P+1}, \tag{3.10}$$

where again there is no summation over i, j from the fourth row. Using the previous result we obtain the expression

$$U(A)_{ij}(z) = A_{ij}\exp\left(\frac{\mathrm{i}}{2\pi\alpha'}(y^I_i - y^I_j)g_{IJ}X^J(z)\right) \tag{3.11}$$

without summation over i, j. We then see that (3.5) has the correct form of a matrix-valued string field for the description of the string configuration in the background (3.1) of N D-instantons.

Before we turn to the next example, we must certainly find some consistency conditions which these generalized conformal operators should obey. We shall proceed as follows. Let us start with the general configuration of N D-instantons described by any $U(N)$-valued matrices Y^I. Then we require that the matrix-valued string field $\hat{\Phi}$ should obey the linearized equation of motion of the string field theory action. In the Abelian case this leads to the requirement that the given state is annihilated by the BRST operator Q_B. It is reasonable to presume that this holds in the non-Abelian case as well, so we obtain the condition

$$Q_B|\hat{\Phi}\rangle = 0, \qquad |\hat{\Phi}\rangle = \Phi(0)U(A)(0)|0\rangle. \tag{3.12}$$

We shall study the consequence of this equation. In order to do this we must find an OPE between various matrix-valued operators and the stress–energy tensor of the open string theory

$$T(z) = -\frac{1}{\alpha'}\partial_z X^I(z)\partial_z X^J(z)g_{IJ}, \qquad X^I(z)X^J(w) = -\tfrac{1}{2}\alpha'\ln(z-w)g^{IJ}, \qquad I, J = 1,\ldots,26. \tag{3.13}$$

Using (3.13) we can easily calculate the OPE between $T(z)$ and $U(0)$. For example, let us consider the OPE between the stress–energy tensor (3.13) and the first two terms in the expansion of $U(0)$ acting on any A corresponding to any CFT operator Φ,

$$T(z)\frac{\mathrm{i}}{2\pi\alpha'}[Y^I,A]g_{IJ}X^J(0) \sim \frac{1}{z}\frac{\mathrm{i}}{2\pi\alpha'}[Y^I,A]g_{IJ}\partial_z X^J(0),$$
$$T(z)\frac{1}{2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^{I_1},[Y^{I_2},A]]g_{I_1J_1}g_{I_2J_2}X^{J_1}(0)X^{J_2}(0) \sim \frac{1}{2z}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^{I_1},[Y^{I_2},A]]g_{I_1J_1}g_{I_2J_2}\partial_z(X^{J_1}(0)X^{J_2}(0)) - \frac{\alpha'}{4z^2}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2[Y^I,[Y^J,A]]g_{IJ}. \tag{3.14}$$

Generally we have

$$
\begin{aligned}
T(z)\frac{1}{P!}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P}&[Y^{I_1},[Y^{I_2},\ldots,[Y^{I_P},A]]]g_{I_1J_1}\ldots g_{I_PJ_P}X^{J_1}(0)\ldots X^{J_P}(0)\\
&\sim\frac{1}{z}\frac{1}{P!}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P}\sum_{k=1}^{P}[Y^{I_1},[Y^{I_2},\ldots,[Y^{I_k},\ldots,[Y^{I_P},A]]]]\\
&\times g_{I_1J_1}\ldots g_{I_kJ_k}\ldots g_{I_PJ_P}X^{J_1}(0)\ldots X^{J_{k-1}}(0)\partial_zX^{J_k}(0)X^{J_{k+1}}(0)\ldots X^{J_P}(0)\\
&-\frac{1}{z^2}\frac{\alpha'}{4P!}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P}\sum_{m=1,n=2,m\neq n}^{P}g_{IJ}g^{IJ_m}g^{JJ_n}\\
&\times[Y^{I_1},[Y^{I_2},\ldots,[Y^{I_m},\ldots,[Y^{I_n},\ldots,[Y^{I_P},A]]]]]g_{I_1J_1}\ldots g_{I_PJ_P}\\
&\times X^{J_1}(0)\ldots X^{J_{m-1}}(0)X^{J_{m+1}}\ldots X^{J_{n-1}}(0)X^{J_{n+1}}(0)\ldots X^{J_P}(0). \qquad (3.15)
\end{aligned}
$$

From the previous expression we can deduce that generally there is no well defined OPE between the stress–energy tensor and the matrix-valued string field. As a consequence of this fact we cannot define how such a matrix-valued string field transforms under conformal transformations and hence we cannot define string field action. In fact, in analogy with the Abelian case we would like to have an OPE in the form

$$T(z)\hat{\Phi}(0)\sim\frac{1}{z^2}h(\hat{\Phi})(0)+\frac{1}{z}\partial_z\hat{\Phi}(0), \qquad (3.16)$$

where $h(\hat{\Phi})$ is a conformal dimension of given field $\hat{\Phi}$. In order to obtain the OPE in a similar form we demand that the background configuration of D-instantons obeys the following rule:

$$[Y^I,[Y^J,B]]-[Y^J,[Y^I,B]]=0, \qquad \forall B,I,J=1,\ldots,26, \qquad (3.17)$$

or equivalently

$$[[Y^I,Y^J],B]=0\Rightarrow[Y^I,Y^J]=\mathrm{i}\theta^{IJ}1_{N\times N}. \qquad (3.18)$$

When we apply the trace operation on the last equation above we see that in the case of finite matrices the only nontrivial solution is $\theta^{IJ}=0$; however, there is a nonzero θ^{IJ} in the case of infinite-dimensional $U(N)$ matrices Y^I as well known from various matrix models (for a review and an extensive list of references, see [33–36]). We can expect that this configuration describes a higher-dimensional D-brane with noncommutative world-volume. In the case of finite-dimensional matrices the only possible solution is

$$[Y^I,Y^J]=0. \qquad (3.19)$$

Conditions (3.18) and (3.19) are precisely solutions of the equation of motion arising from the low-energy action for N D-instantons. We have seen a similar result in our previous paper [22], where the requirement of the nilpotence of the matrix-valued BRST operator leads to the conclusion that the background configuration of D-instantons must obey equations (3.18) and (3.19).

Using (3.17) we can move Y^{I_m} and then Y^{I_n} to the left-hand side of the second expression in (3.15). Since we have P possible I and $P-1$ J and they all appear in the expression in a symmetric way, the summation in the second expression in (3.15) gives the factor $P(P-1)$, so the second term in (3.15) gives

$$
\begin{aligned}
-\frac{\alpha'P(P-1)}{4P!z^2}&\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P}g_{IJ}[Y^I,[Y^J,[Y^{I_1},\ldots,[Y^{I_{P-2}},A]]]]\\
&\times g_{I_1J_1}\ldots g_{I_{P-2}J_{P-2}}X^{J_2}(0)\ldots X^{J_{P-2}}(0)\\
&=-\frac{\alpha'}{4z^2(P-2)!}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{2}g_{IJ}\left[Y^I,\left[Y^J,\left(\frac{\mathrm{i}}{2\pi\alpha'}[Y^K,\cdot]g_{KL}X^L(0)\right)^{P-2}A\right]\right] \qquad (3.20)
\end{aligned}
$$

and the first equation in (3.15) gives

$$\frac{1}{z}\frac{1}{P!}\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^{P}\sum_{k=1}^{P}[Y^{I_1},[\ldots,[Y^{I_k},[\ldots,[Y^{I_P},A]]]]]$$
$$\times\, g_{I_1J_1}\ldots g_{I_kJ_k}\ldots g_{I_PJ_P}X^{J_1}(0)\ldots X^{J_{k-1}}(0)\partial_z X^{J_k}(0)X^{J_{k+1}}(0)\ldots X^{J_P}(0)$$
$$=\frac{1}{zP!}\partial_z\left(\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)[Y^I,\cdot\,]g_{IJ}X^J(0)\right)^{P}(A). \tag{3.21}$$

Collecting all previous results we obtain the following OPE:

$$T(z)\hat{\Phi}(0)\sim\frac{1}{z^2}\left(\frac{1}{16\pi^2\alpha'}g_{IJ}[Y^I,[Y^J,\hat{\Phi}(0)]]+h_\Phi\hat{\Phi}(0)\right)+\frac{1}{z}\partial_z\hat{\Phi}(0), \tag{3.22}$$

where h_Φ is the conformal dimension of the operator $\Phi(0)$ in (3.5). We see that 'the conformal dimension' of $\hat{\Phi}$ is now matrix valued and depends on the configuration of various D-instantons. Since the ghost sector does not depend on the background configuration of D-instantons, the action of the BRST operator on Φ is the same as in the Abelian case. When we also use the gauge

$$b_0|\Phi\rangle=0 \tag{3.23}$$

we see that the linearized equation of motion (3.12) leads to the condition

$$\frac{1}{16\pi^2\alpha'}g_{IJ}[Y^I,[Y^J,\hat{\Phi}(0)]]+h_\Phi\hat{\Phi}(0)=0, \tag{3.24}$$

where (in gauge $b_0|\Phi\rangle=0$)

$$Q_B|\Phi\rangle=h_\Phi|\Phi\rangle. \tag{3.25}$$

Condition (3.24) expresses the fact that each component $\hat{\Phi}_{ij}$ of the matrix-valued string field describes the state of the open string connecting the ith D-instanton with the jth D-instanton which is on the mass shell. For example, for the diagonal background (3.1) the first term in the bracket in (3.22) gives

$$\frac{1}{16\pi^2\alpha'}g_{IJ}[Y^I,[Y^J,\hat{\Phi}(0)]]_{ij}=\frac{1}{16\pi^2\alpha'}g_{IJ}[y_i^I\delta_{ik}[Y^J,\hat{\Phi}(0)]_{kj}-[Y^J,\hat{\Phi}(0)]_{ik}\delta_{kj}y_j^I]$$
$$=\frac{1}{16\pi^2\alpha'}g_{IJ}(y_i^I-y_j^I)(y_i^J-y_j^J)\hat{\Phi}_{ij}(0) \tag{3.26}$$

(again no summation over i, j). Then we have a natural result that the conformal dimension of each component $\hat{\Phi}_{ij}$ is proportional to the distance between the ith D-instanton and the jth D-instanton.

The OPE between the generalized matrix-valued string field and the stress–energy tensor also has an important consequence for the conformal transformation of the given string field and hence for the generalized form of the string field action. Recall that a primary vertex operator of conformal dimension h transforms under $z'=f(z)$ as

$$\mathcal{O}'(z')=\left(\frac{\partial f}{\partial z}\right)^{-h}\mathcal{O}(z)=\exp\{(-h\ln(f'(z))\}\mathcal{O}(z). \tag{3.27}$$

From the second form of this description and from the fact that for the matrix-valued string field the first term in (3.22) acts as a matrix on the given string field we can anticipate the following generalized matrix-valued conformal transformation:

$$\hat{\Phi}'(z')=\exp\left(-\ln(f(z))\left[\frac{1}{16\pi^2\alpha'}g_{IJ}[Y^I,[Y^J,\cdot]]+h_\Phi\right]\right)(\hat{\Phi})(z), \tag{3.28}$$

where the exponential function should be understood as a matrix-valued function and its action on $\hat{\Phi}(z)$ in the form of a Taylor expansion and where h_Φ is the conformal dimension of the operator $\Phi(z)$. In particular, for the background (3.1) we have

$$\hat{\Phi}(z)_{ij} = A_{ij} \exp\left(\frac{\mathrm{i}}{2\pi\alpha'}(y_i^I - y_j^I) g_{IJ} X^J(z)\right). \tag{3.29}$$

In order to determine the behaviour of this field under conformal transformation, we must expand the exponential function in (3.28) and let it act on $\hat{\Phi}$. For example, for the background (3.1) we obtain the result that the field $\hat{\Phi}_{ij}$ that describes the state of the string going from the ith D-instanton to the jth D-instanton transforms under the general conformal transformation according to the usual rule

$$\hat{\Phi}'(z')_{ij} = \left(\frac{\mathrm{d}f(z)}{\mathrm{d}z}\right)^{-\frac{1}{16\pi^2\alpha'}(y_i - y_j)^2 - h_\Phi} \hat{\Phi}(z)_{ij}. \tag{3.30}$$

4. Vertex operators for configuration of $N \to \infty$ D-instantons with nonzero θ^{IJ}

Now we turn to the second example, which is the background configuration of N D-instantons (in this case $N \to \infty$) in the form

$$[Y^a, Y^b] = \mathrm{i}\theta^{ab}, \qquad a, b = 1, \ldots, 2p, \qquad Y^m = 0, \qquad m = 2p+1, \ldots, 26. \tag{4.1}$$

As in the previous case we begin with (3.6), where the second term is proportional to

$$\mathrm{i}[Y^I, A] g_{IJ} X^J(0). \tag{4.2}$$

Following [27–29] we introduce the set of matrices

$$O_k = \mathrm{e}^{\mathrm{i}\theta^{ij} k_i p_j}, \qquad p_b = \theta_{bc} Y^c, \qquad \theta_{ac}\theta^{cb} = \delta_a^b. \tag{4.3}$$

Then we can write any matrix as follows:

$$A = \int \mathrm{d}^{2p}k \, \exp[\mathrm{i}\theta^{ab} k_a p_b] A(k), \tag{4.4}$$

where $A(k)$ is an ordinary function. Then it is easy to see [28, 29]

$$[p_i, O_k] = k_i O_k, \qquad [p_i, p_j] = -\mathrm{i}\theta_{ij} \tag{4.5}$$

and consequently

$$\begin{aligned}
&\frac{2\pi}{4\pi^2\alpha'}[Y^a, A] g_{ab} X^b(0) = [p_a, A] G^{ab} \tilde{X}_b(0) = \int \mathrm{d}^{2p}k \, k_a G^{ab} \tilde{X}_b(0) A(k) O_k, \\
&G^{ab} = -\frac{1}{4\pi^2\alpha'^2}\theta^{ac} g_{cd} \theta^{db}, \; \tilde{X}_b(0) = 2\pi\alpha' \theta_{bc} X^c(0), \\
&\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^P [Y^{I_1}, [Y^{I_2}, \ldots, [Y^{I_P}, A]]] g_{I_1 J_1} g_{I_2 J_2} \ldots g_{I_P J_P} X^{J_1}(z) X^{J_2}(z) \ldots X^{J_P}(z) \\
&\qquad = \mathrm{i}^P \int \mathrm{d}^{2p}k \, k_{a_1} G^{a_1 b_1} \tilde{X}_{b_1}(z) \ldots k_{a_P} G^{a_P b_P} \tilde{X}_{b_P}(z) A(k) O_k.
\end{aligned} \tag{4.6}$$

Then it follows that

$$U(A)(z) = \int \mathrm{d}^{2p}k \, \exp(\mathrm{i}k\tilde{X}(z)) A(k) O_k, \qquad k\tilde{X} = k_a G^{ab} \tilde{X}_b \tag{4.7}$$

and consequently for any CFT operator $\Phi(z)$ (corresponding to some particular A) we obtain the matrix-valued string field $\hat{\Phi}$

$$\hat{\Phi} = U(A)\Phi(z) = \int \mathrm{d}^{2p}k \, \Phi(z) \exp(\mathrm{i}k\tilde{X}(z)) A(k) O_k. \tag{4.8}$$

We define generalized matrix-valued vertex operators the form of which we can deduce from (4.8),

$$V(k,\Phi(z)) = \Phi(z)\exp(\mathrm{i}k\tilde{X}(z))O_k. \tag{4.9}$$

It is important to include the matrix O_k in the definition of the matrix-valued vertex operator V in order to stress its matrix nature, since any correlation function of these operators contains the trace over matrix indices. Let us consider two such matrix-valued vertex operators,

$$V(k_1,\Phi)(z) = O_{k_1}\Phi(z)\mathrm{e}^{\mathrm{i}k_1X(z)}, \qquad V(k_2,\Psi)(z) = O_{k_2}\Psi(z)\mathrm{e}^{\mathrm{i}k_2X(z)}, \tag{4.10}$$

which should appear in the calculation of the correlation function and in particular in the string field theory action. Let us calculate the generalized OPE of these two operators, where we include the matrix multiplication. In fact, the calculation of the OPE is an easy task. Matrix multiplication only affects the parts containing O_{k_1}, O_{k_2}, that gives

$$O_{k_1}O_{k_2} = \exp(\mathrm{i}\theta^{ij}(k_1+k_2)_i p_j - \tfrac{1}{2}\mathrm{i}\theta^{ab}k_ak_b) = \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{ab}k_ak_b}O_{k_1+k_2} \tag{4.11}$$

using

$$[\mathrm{i}\theta^{ab}k_ap_b, \mathrm{i}\theta^{cd}k_cp_d] = -\theta^{ab}\theta^{cd}k_ak_c(-\mathrm{i}\theta_{bd}) = -\mathrm{i}\theta^{ab}k_ak_b \tag{4.12}$$

and also using the relation

$$\mathrm{e}^A\mathrm{e}^B = \mathrm{e}^{A+B+\frac{1}{2}[A,B]} \tag{4.13}$$

that is valid for operators whose commutator is a pure number. Then we have

$$\begin{aligned} V(k_1,\Phi)(z)V(k_2,\Psi)(w) &= O_{k_1}\Phi(z)\mathrm{e}^{\mathrm{i}k_1\tilde{X}(z)}O_{k_2}\Psi(w)\mathrm{e}^{\mathrm{i}k_2\tilde{X}(w)} \sim \mathrm{e}^{-\frac{\mathrm{i}}{2}\theta^{ab}k_ak_b}O_{k_1+k_2} \\ &\quad\times ((z-w)^{\frac{\alpha'}{2}k_{1a}G^{ab}k_{2b}}\exp(\mathrm{i}(k_1+k_2)\tilde{X}(w))\Phi(w)\Psi(w)+\cdots) \end{aligned} \tag{4.14}$$

where the dots mean other possible singular terms arising from the expansion of $\mathrm{e}^{\mathrm{i}k\tilde{X}(z)}$ and from the OPE between $\Phi(z)$ and $\Psi(w)$. We see that the previous OPE has the same form as the OPE of the vertex operators in the presence of the background field $B_{ab} = (\frac{1}{\theta})_{ab}$ as is well known from the seminal paper [25]. It is also important to stress that thanks to the redefinition $X^c(z) = \frac{1}{2\pi\alpha'}\theta^{cd}\tilde{X}_d(z)$ the stress–energy tensor appears as

$$\begin{aligned} T(z) &= -\frac{1}{\alpha'}\partial X(z)^I\partial X(z)^J g_{IJ} = -\frac{1}{\alpha'}\left(\frac{1}{4\pi^2\alpha'}\theta^{ac}\tilde{X}_c(z)\theta^{bd}\tilde{X}_d g_{ab}\right) - \frac{1}{\alpha'}\partial X^i(z)\partial X^j(z)g_{ij} \\ &= -\frac{1}{\alpha'}\partial\tilde{X}_a(z)\partial\tilde{X}_b(z)G^{ab} - \frac{1}{\alpha'}\partial X^i(z)\partial X^j(z)g_{ij}. \end{aligned} \tag{4.15}$$

In other words, the world-sheet stress–energy tensor is expressed in terms of the open string metric G_{ab} in dimensions labelled with $\tilde{X}^a, \tilde{X}^b, \ldots$, hence the OPE between the part of the stress–energy tensor depending on the open string metric and any matrix-valued vertex operator is a function of the open string quantities only, again in agreement with [25].

We should also study the generalized conformal transformation (3.28)

$$V'(k,\Phi,z') = \exp\left(-\ln(f(z))\left[\frac{1}{16\pi^2\alpha'}g_{IJ}[Y^I,[Y^J,\cdot]] + h_\Phi\right]\right)V(k,\Phi,z). \tag{4.16}$$

In fact, h_Φ is given solely by the conformal dimension of $\Phi(z)$ and the matrix multiplication defined in the exponential function in (4.16) acts on O_k only. Then we can expand the exponential function and use (4.5). It is easy to see that we obtain the standard conformal transformation of the vertex operator with the momentum k_a

$$V'(k,\Phi,z') = \exp\left(-\ln(f(z))\left[\frac{\alpha'}{4}k^2 + h_\Phi\right]\right)V(k,\Phi,z) = \left(\frac{\mathrm{d}f(z)}{\mathrm{d}z}\right)^{-\frac{\alpha' k^2}{4} - h_\Phi}V(k,\Phi,z) \tag{4.17}$$

with $k^2 = k_a G^{ab} k_b$. From this expression we see that $V(k, \Phi, z)$ has the conformal dimension equal to $\alpha' k^2/4 + h_\Phi$ as we could expect. We can also calculate the OPE between the stress–energy tensor and matrix-valued vertex operators. Since the OPE between the vertex operator and the stress–energy tensor determines the conformal dimension of the given operator and this is known from (4.17), we do not need to work out this OPE and can determine its form directly from (4.17).

5. A more general example

As the last example, let us consider the background configuration of D-instantons in the form

$$Y^a = 1_{N\times N} \otimes y^a, \qquad Y^i = \begin{pmatrix} y_1^i \otimes 1 & 0 & \dots & 0 \\ 0 & y_2^i \otimes 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & y_N^i \otimes 1 \end{pmatrix}, \qquad i = 2p+1, \dots, 26, \tag{5.1}$$

where

$$[y^a, y^b] = \mathrm{i}\theta^{ab}, \qquad a, b = 1, \dots, 2p \tag{5.2}$$

are infinite-dimensional matrices. It is easy to see that this configuration obeys (3.18) and hence corresponds to the consistent background configuration. Now we shall write any matrix A as follows:

$$A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \dots & \dots & \dots & \dots \\ A_{N1} & \dots & \dots & A_{NN} \end{pmatrix}, \tag{5.3}$$

where $A_{xy}, x, y = 1, \dots, N$ are infinite-dimensional matrices. Let us write any A_{xy} in the form

$$A_{xy} = \int \mathrm{d}^{2p} k_{xy}\, A_{xy}(k_{xy}) \exp(\mathrm{i}\theta^{ab} k_{axy} p_b). \tag{5.4}$$

Now we can write

$$\begin{aligned} \frac{\mathrm{i}}{2\pi\alpha'}[Y^I, A]_{xy} g_{IJ} X^J(z) &= \frac{\mathrm{i}}{2\pi\alpha'}[\delta_{xz} \otimes y^a, A_{zy}] g_{ab} X^b(z) + \frac{\mathrm{i}}{2\pi\alpha'}[y_x^i \delta_{xz} \otimes 1, A_{zy}] g_{ij} X^j(z) \\ &= [p_a, A]_{xy} G^{ab} \tilde{X}_b(z) + \frac{\mathrm{i}}{2\pi\alpha'}(y_x^i - y_y^i) g_{ij} X^j(z) A_{xy} \\ &= \mathrm{i} \int \mathrm{d}^{2p} k_{xy} \left(k_{xya} G^{ab} \tilde{X}_b(z) + \frac{1}{2\pi\alpha'}(y_x^i - y_y^i) g_{ij} X^j(z) \right) A_{xy}(k_{xy}) O_{k_{xy}}. \end{aligned} \tag{5.5}$$

The second term in (3.6) gives

$$\begin{aligned} &\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^2 [Y^I, [Y^J, A]] g_{IK} g_{KL} X^K(z) X^L(z) \\ &\quad = \mathrm{i}^2 \int \mathrm{d}^{2p} k_{xy}\, k_{xya_1} G^{a_1 b_1} \tilde{X}_{b_1}(z) k_{xya_2} G^{a_2 b_2} \tilde{X}_{b_2}(z) A_{xy}(k_{xy}) O_{k_{xy}} \\ &\qquad + \mathrm{i}^2 \int \mathrm{d}^{2p} k_{xy} A(k_{xy})_{xy} \left(\frac{1}{2\pi\alpha'}(y_x^i - y_y^i) g_{ij} X^j(z) \right)^2 \\ &\qquad + 2\mathrm{i}^2 \int \mathrm{d}^{2p} k_{xy}\, O_{k_{xy}} k_{xya} G^{ab} \tilde{X}_b(z) \frac{1}{2\pi\alpha'}(y_x^i - y_y^i) g_{ij} X^j(z) \\ &\quad = \mathrm{i}^2 \int \mathrm{d}^{2p} k_{xy}\, O_{k_{xy}} A(k_{xy})_{xy} \left(k_{axy} G^{ab} \tilde{X}_b(z) + \frac{1}{2\pi\alpha'}(y_x^i - y_y^i) g_{ij} X^j(z) \right)^2. \end{aligned} \tag{5.6}$$

Generally, we have

$$\left(\frac{\mathrm{i}}{2\pi\alpha'}\right)^P [Y^{I_1},[Y^{I_2},\ldots,[Y^{I_P},A]]]_{xy} g_{I_1J_1}g_{I_2J_2}\ldots g_{I_PJ_P}X^{J_1}(z)X^{J_2}(z)\ldots X^{J_P}(z)$$

$$= \mathrm{i}^P \int \mathrm{d}^{2p}k_{xy}\left(k_{xya}G^{ab}\tilde{X}_b(z) + \frac{1}{2\pi\alpha'}(y_x^i - y_y^i)g_{ij}X^j(z)\right)^P A(k_{xy})_{xy}O_{k_{xy}}. \quad (5.7)$$

Using these results we can write the generalized matrix-valued string field (3.5) in the form

$$\hat{\Phi}_{xy}(z) = \int \mathrm{d}^{2p}k_{xy}\, A(k_{xy})_{xy}\exp\left(\mathrm{i}k_{xy}\tilde{X}(z) + \frac{\mathrm{i}}{2\pi\alpha'}(y_x^i - y_y^i)g_{ij}X^j(z)\right)O_{k_{xy}}. \quad (5.8)$$

In summary, we have obtained the matrix-valued string field for configuration of N D$2p$-branes with the noncommutative world-volume in dimensions labelled with x^a, $a = 1,\ldots,2p$, that are placed in the different transverse positions labelled with y_x^i, $i = 2p+1,\ldots,26$, $x = 1,\ldots,N$.

We hope that the three examples given above sufficiently support our proposed form of the generalized matrix string field (3.5). Then we propose that the string field theory action for any configuration of N D-instantons obeying (3.18) has the form

$$S = \frac{1}{g_0^2}\mathrm{Tr}\left(\frac{1}{2\alpha'}\langle I\circ\hat{\Phi}(0)Q_B\hat{\Phi}(0)\rangle + \frac{1}{3}\langle f_1\circ\hat{\Phi}(0)f_2\circ\hat{\Phi}(0)f_3\circ\hat{\Phi}(0)\rangle\right), \quad (5.9)$$

where now conformal transformations $I\circ\hat{\Phi}(0)$, $f_i\circ\hat{\Phi}(0)$, $i = 1,2,3$, are defined by (3.28). The precise study of this action and its particular solutions will be performed in the forthcoming work.

6. Conclusion

In this paper we have proposed the matrix-valued form of the string field, that could be useful for description of D-instanton configuration using the string field theory action written in the CFT language [23, 24]. We have calculated the OPE of these matrix-valued string fields with the stress–energy tensor. We have seen that the condition that the OPE is well defined leads to the requirement that the background configuration of D-instantons should obey the equation that can be interpreted as the equation of motion arising from the low-energy matrix theory action. We have also proposed the generalized conformal transformation of the matrix-valued string fields.

As a next step of our research we shall study the proposed matrix-valued string field theory action (5.9). We shall also extend this approach to the supersymmetric case.

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